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EXPERIMENTAL AND COMPUTATIONAL STUDY OF THE EFFECT OF THE SYSTEM SIZE ON ROUGH SURFACES FORMED BY SEDIMENTING PARTICLES IN QUASI-TWO-DIMENSIONS

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The roughness exponent of surfaces obtained by dispersing silica spheres into a quasi-two-dimensional cell is examined using experimental and computational methods. The cell consists of two glass plates separated by a gap, which is comparable in size to the diameter of the beads. We have studied the effect of changing the gap between the plates to a limit of about twice the diameter of the beads. If the conventional scaling analysis is performed, the roughness exponent is found to be robust against changes in the gap between the plates. The surfaces formed have two roughness exponents in two length scales, which have a crossover length about 1 cm.; however, the computational results do not show the same crossover behavior. The single exponent obtained from the simulations stays between the two roughness exponents obtained in the experiments.

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I. INTRODUCTION

The formation of rough surfaces through the sedimentation of particles through a viscous fluid is a complex problem, but one with many applications, ranging from the study of fundamental non-equilibrium statistical physics to various industrial processes such as the growth of films by deposition [1, 2]. The presence of the viscous fluid allows both particle/particle interactions as well as particle/wall interactions during sedimentation which are not normally considered in deposition processes, but which may well be present in actual systems of interest and which it can be assumed will have an effect on the final surface. Surfaces formed by sedimentation are close to the original problem of sedimentation of particles sedimenting along straight vertical trajectories first studied by Edwards and Wilkinson [3]. However, the hydrodynamic particle/particle and particle/wall forces are in principle long-range, making the rough surfaces formed by particles sedimenting in a viscous fluid are a different growth situation from the simpler vertical deposition. The situation is further complicated by the presence of back flows of fluid caused by the motion of significant numbers of particles [4, 5, 6].

In this work we are primarily interested studying the effect of the particle-wall interactions on the roughness of the final interface. The problem of the motion of a sphere parallel to a single wall as the limiting case of motion of a small sphere in a cylindrical container when the sphere approaches the cylinder wall and the more general one of the motion of a sphere parallel to two external walls were treated by Faxen [7]. Unfortunately, the exact nature of the interactions between the particles in the presence of the walls is very difficult to determine. Analytical sedimentation theory has succeeded in analyzing the effective settling velocity [8, 9, 10, 11, 12, 13, 14] and velocity fluctuations [15, 16] of particles in a dilute regime in the presence of the walls and some features of many-body interactions between the particles [17, 18] when there are no walls [19, 20, 21, 22]. Recent theoretical [5, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33] and experimental [34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44] work hold out some hope of determining effective particle interactions through a wide range of volume fractions and Peclet numbers in sedimentation problems. Also simulations of deposition of elongated particles [45, 46, 47, 48, 49, 50] indicate that application of sedimentation problems is not restricted to spherical particles, but may well expand into areas like the paper industry. New developments in the effect of the container size on the divergence of the velocity fluctuations [51, 52, 53, 54, 55, 56] show the importance of the presence of the wall in determining the particle-particle and particle-wall interactions.

In our previous work [57, 58, 59, 60] on the quasi-one-dimensional surfaces formed by particles sedimenting through a viscous fluid in a quasi-two-dimensional cell we have found that the surfaces formed by sedimenting particles are rough on all length scales between the particle size and the cell size. Using the scaling ansatz proposed by Family and Vicsek [61] discussed below, it was found that different roughness exponents were found in two different length-scale regimes, with a crossover length scale. These roughness exponents and the crossover length scale have been found to be independent of the cell aspect ratio or the viscosity of the fluid through which the particles settle [58]. The exponent found at long length scales has been shown to depend on the rate at which particles are deposited into the cell (hence to the strength of the interaction between the particles) [59]. This lead to the conclusion that the scaling exponent seen at long length scales depended on the details of the hydrodynamic interactions between the particles, while the exponent seen at

small length scales, which remained relatively unaffected by changes in the deposition rate, may be due to more universal considerations.

In this work, we have investigated the effect of the particle-wall interactions on the roughness of the final interfaces formed by quasi-two dimensional sedimentation of small glass beads through a viscous fluid. Simulations of the same system are compared to experimental results with the aim of untangling the effects of the viscous fluid on the process from the better understood effects of the deposition process.

II. EXPERIMENTAL WORK

In our previous work [57, 58, 59, 60], sedimentation experiments in quasi-two dimensions have been carried out using two different types of cells, denoted as "closed" and "open" cells. Closed cells were constructed of 1/4 in. float glass, held 1 mm apart by sealed side frames of precision machined Plexiglas. Around 10,000 0.6 mm-diameter monodisperse silica spheres were placed in the cell, which was then filled with a viscous fluid (such as glycerin or paraffin oil) and closed. Each cell could be rotated about a horizontal axis perpendicular to the gap direction. When the cell was rotated, the particles which had been at rest on the bottom fell though the viscous fluid, slowly building up a new surface at the bottom of the cell. In the closed cells we only had a fixed gap size between the cell walls, but we had different sizes of cells filled with fluids of different viscosity. However the number of the particles sedimenting at any time could not be controlled. The open cell (which was the one used in this experimental work) was also constructed of 1/4 in. float glass, separated by strips of Teflon of known thickness. It had dimensions comparable to the closed cell, but was open at the top, so that beads could be dispersed through a funnel which steadily dropped beads as it traveled back and forth across the top of the cell (Fig. 1). In this way, the deposition rate of the beads into the cell could be controlled precisely by varying the speed and the size of the funnel. The cell could be taken apart and a different thickness of Teflon inserted to change the gap between the plates.

We have tested the effect of changing the distance between the walls on the roughness of the final interface while keeping the particle density fairly constant. The experiments took place in the open cell and we investigated the effect of variability in the gap by setting the gap at different values, and measuring the effect of the walls on the roughness of the final interface formed during sedimentation. We investigated gaps ranging from 0.8 mm to 2.0 mm, while the particle size was kept constant at 0.6 mm. The ratio of the gap thickness to the bead diameter was defined as a dimensionless parameter R, and our experiments spanned a gap/bead diameter ratio of $R = 1.33$ to $R = 3.33$. In all cases, the deposition rate of beads into the cell was controlled at about 4 beads/sec so the average distance between the particles was about $20R$. The surface was photographed during and at the end of the deposition process and the photographs were digitized by a Nikon LS-2000 film scanner. Individual particles were typically resolvable and thus the position of the particles on the interface could be traced accurately (Fig. 2). There is a limit to the extent over which the gap can be widened without changing the method of analysis, since at one point it will no longer be possible to analyze the rough surface as a one-dimensional interface. We believe that we are already past that limit at $R = 2$, but to give an estimate of the effects of the wall separation to the interested reader we have included the data for $R > 2$.

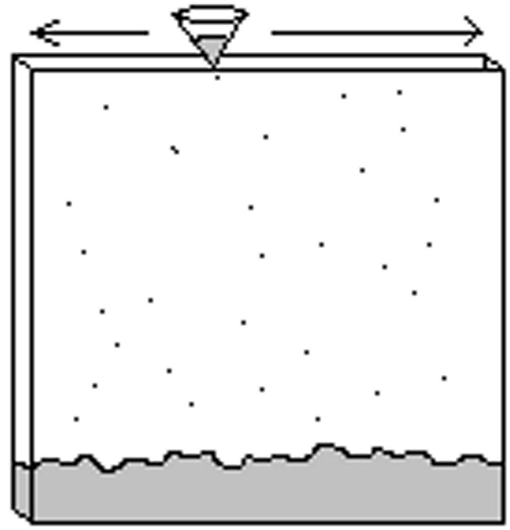

FIG. 1: The sedimentation cell consists of two glass plates with a small gap between them, of the order of the diameter of the glass beads. The cell is filled with a viscous fluid such as oil, and a funnel sweeps across the top of the cell, delivering a mixture of oil and beads to the cell. The beads settle to the bottom of the cell and build a rough surface.

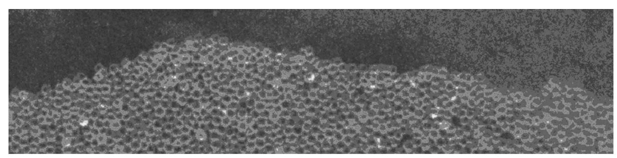

FIG. 2: Portion of a rough surface formed by 0.6 mm diameter glass beads sedimenting through heavy paraffin oil.

III. SIMULATIONS

We have also carried out computer simulations to investigate the effect of changing the cell width to the properties of rough surfaces formed by sedimentation. During the simulations we have deposited particles onto a quasi-

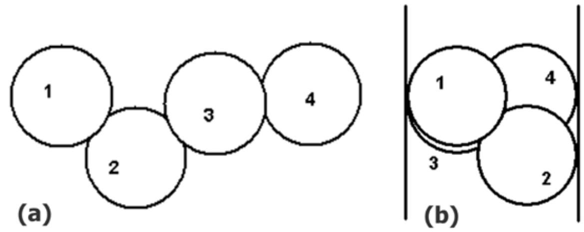

FIG. 3: A sample of the final surface obtained for $R = 1.6$. We have not shown the particles underneath the uppermost particles. (a) Front view, (b) side view of the deposited particles.

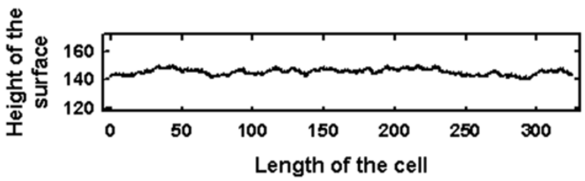

FIG. 4: The final surface after 50000 particles are deposited for the case $R = 1.1$. The length and the height of the surface are in units of R.

two dimensional surfaces bounded by two walls. The separation between the walls (the width of the cell) is varied between $R = 1.1$ and $R = 1.9$. At the two ends of the cell we have used periodic boundary conditions. The particles are dropped onto the surface at random locations and once they touched the surface, they rolled to the local minimum which is reached when they are in contact with two other particles and a wall. A sample of the final surface is shown in (Fig. 3).

During the different runs we did not fix the length of the cell, but we varied the number of particles deposited with the requirement that the length of the final surface was about twice as large as the height. We have deposited 100 surfaces of 5000, 10000, 50000, and 100000 particles each. We have observed that the final results did not significantly depend on the number of particles deposited. Figure 4 shows an example of the final surface deposited.

IV. DISCUSSION

As in the previous work, we have analyzed these rough surfaces using the scaling ansatz proposed by Family and Vicsek [61]. In this ansatz, the rms thickness of the interface is defined to be:

$$W(L,t) = \left[\frac{1}{N}\sum_{i=1}^{N}\tilde{h}(x_i,t)^2\right]^{\frac{1}{2}} \tag{1}$$

where

$$\tilde{h}(x_i,t) = h(x_i,t) - \bar{h}(t) \tag{2}$$

and

$$\bar{h}(t) = \frac{1}{N}\sum_{i=1}^{N} h(x_i,t). \tag{3}$$

As discussed in the previous work, it is not at all clear that our system is in a scaling regime, nor is it obvious that scaling ideas should apply to sedimentation, but a useful way of analyzing our data is to adopt and extend the standard roughness analysis by tentatively accepting a scaling ansatz for rough interface growth. If we follow this ansatz, we expect that:

$$W(L,t) = L^{\alpha} f(t/L^{\alpha/\beta}) \tag{4}$$

where the exponents α and β are the static and dynamic scaling exponents. The function $f(t/L^{\alpha/\beta})$ is expected to have an asymptotic form such that

$$W(L,t) \sim t^{\beta}\ for\ t \ll L^{\alpha/\beta} \tag{5}$$

and

$$W(L,t) \sim L^{\alpha}\ for\ t \gg L^{\alpha/\beta} \tag{6}$$

Fig. 5 shows an example of $W(L,t)$ at a typical gap/bead ratio in the experiments. To minimize the wall effects at the horizontal edges, we have used only the middle 70% of each interface for our analysis.

Again following the scaling ansatz, we find that at all values of R (gap/bead ratio) studied, we still see two roughness exponents in the experiments, where α_s denotes the roughness exponent found at short length scales, while α_l denotes the roughness exponent found at long length scales. These roughness exponents have a crossover length scale at about 1 cm, which is typical from the previous work. Our earlier work corresponded to a gap width of 1.0 mm, and the experimental work gives similar results at this gap width as expected. As the value of R is increased (Fig. 6), we do not see any significant change in the value of either exponent. The simulation data on the other hand shows no crossover and yields one single roughness exponent. Just in the experiments we do not observe a change with changing gap/bead ratio, but the value of the scaling exponent obtained from the simulations is always in between the two exponents obtained from the experiments.

We must note that all of the previous discussion is based on the acceptance of the scaling arguments for this system. A careful review of the experimental data shows a crossover, but the two scaling regimes are not clearly linear (Fig. 5(a)). An alternate argument can be made that this data does not show clear evidence of scaling. If there is scaling, there are two exponents, but the crossover between the two length scales does not appear to be sharply defined. Although it is clear that the behavior at small length scales seems different than that at large

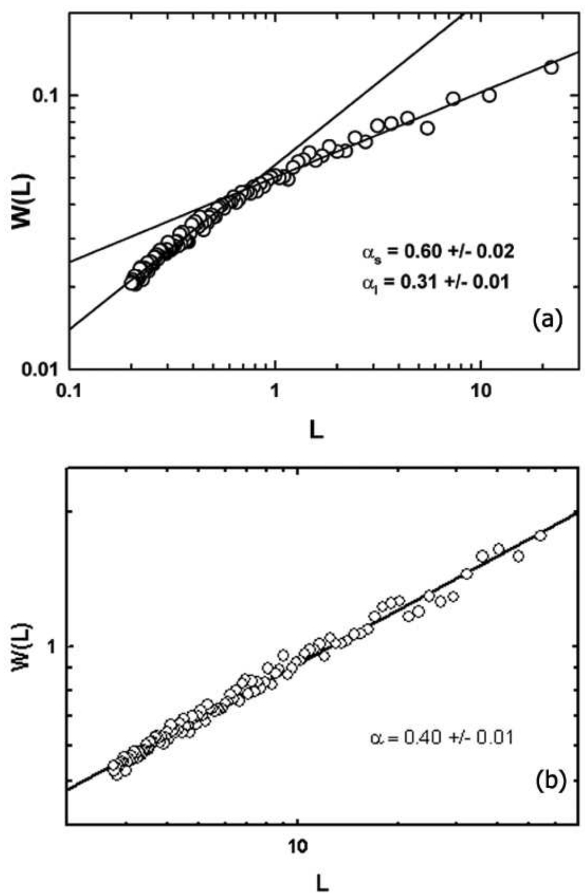

FIG. 5: Roughness function versus L for a typical interface (a) at a gap of 0.8 mm in the experiment and (b) at $R = 1.6$ in the simulation.

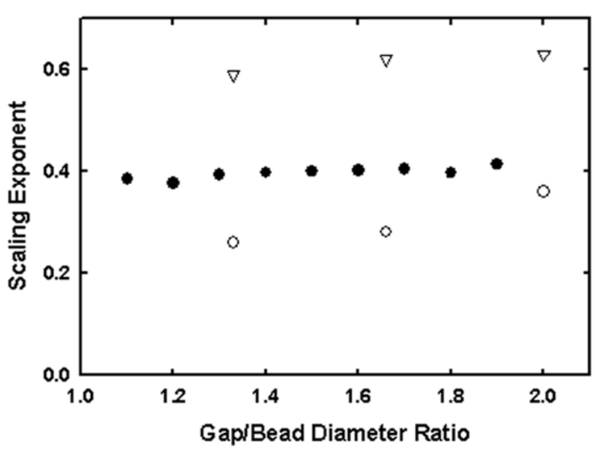

FIG. 6: The change of the average scaling exponents with gap/bead ratio, R. The empty circles (α_s) and empty triangles (α_s) denote the scaling exponents from the experiments.

length scales, an argument can certainly be made that the data change continuously at different length scales. Therefore the data in the simulations show a clearer picture as being the mathematical equivalent of the experimental problem. It has already been noted that even in the absence of particle-particle interactions, particle-wall interactions can play a significant role in determining the final structure of the surface. Also we have ignored effects coming from weight of the particles and also their kinetic energy. In the simulations the particles stopped when they came into contact with two other particles and a wall whereas in the experiments we have observed that some particles did not stay at the local minima and continued towards a global minimum with the effect of their momentum. Thus the simulations should be regarded as a limiting case of infinite viscosity and zero downward momentum.

We have investigated the effect of the interaction between the walls of the container and the sedimenting particles on the roughness exponent of the surface formed by this quasi-two-dimensional sedimentation. If the scaling ansatz is accepted, the roughness exponent is found to be robust to the changes in the separation between the walls of the container as observed in the experiments and by simulating the same problem. We have been unable to reproduce the slight increase in the roughness exponent at $R = 2$ using computational methods. As the experimental data show evidence of continuous change with the lengthscale L, the possibility that scaling arguments do not hold should be taken seriously. In contrast, the simulation data show good agreement with the scaling arguments suggesting that particle-wall interactions can be blamed for the deviation from the scaling behavior.

Acknowledgments

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